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Supply chain coordination for fuzzy random newsboy problem with imperfect quality

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ABSTRACT

Facing to imperfect quality and fuzzy random market demand in the real-life inventory management, a two-echelon supply chain system with one retailer and one manufacturer for perishable products is considered. Two fuzzy random models for the newsboy problem with imperfect quality in the decentralized and centralized systems are presented. The expectation theory and signed distance are employed to transform the fuzzy random model into crisp model. The optimal policies in the two decision-making systems are derived and analyzed contrastively. The theoretical analysis shows that manufacturer's repurchase strategy can achieve the increase in the whole supply chain profit. The influence of the fuzzy randomness of the demand and the defective rate on the optimal order quantity, the whole supply chain profit and the repurchasing price is analyzed via numerical examples.

1. Introduction

Newsboy problems play an important role in the policy making for the single-period inventory problems such as perishable goods, spare parts and seasonal products. Therefore, the application of newsboy problems has triggered considerable attention.

In the classical newsboy problems, several probabilistic inventory models with random demand have been studied in the literature (see, for example [25,14,18,20,21]). Attention has been focused on the randomness aspect of uncertainty and random inventory models are developed by using the probability theory. It is well known that the usage of the probability theory needs a great deal of evidences recorded in the past. However, the historical data are not always available or reliable due to market turbulence or technological innovation. Moreover, the demand becomes extremely variable because of shorter product life cycles in the highly competitive market. Hence, the traditional probability theory and statistical method cannot be used properly to describe this kind of uncertainty and the fuzzy theory is employed to deal with these cases. Depending on the manager's judgments or experiences, the uncertainties and imprecision of data are described by linguistic terms such as "low", "high", "the demand is about d", that is, fuzzy variables. A considerable amount of research has been accomplished on fuzzy newsboy problem. Petrovic et al. [17] presented two fuzzy models for the newsboy problem with fuzzy demand and fuzzy costs. The optimal order quantity is derived by minimizing the total cost in the fuzzy sense. Ishii and Konno [9] modified the traditional discrete random newsboy problem by incorporating the fuzziness of shortage cost explicitly. An optimal ordering quantity realizing the fuzzy max order of the profit function (fuzzy min order considering the profit function) is derived and compared with the optimal ordering quantity of the non-fuzzy newsboy problem. Li et al. [15] considered two models for single-period inventory problem, in one the demand is characterized by a probability distribution while the holding and shortage cost are fuzzy and in the other the costs are crisp but the demand is a fuzzy number. Kao and

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Hsu [12] constructed the newsboy problem in uncertain environment with a method for ranking fuzzy numbers which is adopted to find the optimal order quantity in terms of the cost. Shao and Ji [19] developed the model for multi-product newsboy problem with fuzzy demands and budget constraint. Dutta et al. [7] dealt with the profit maximization under a single-period framework with fuzzy demand and the reordering strategy. Dutta and Chakraborty [8] incorporated the opportunity for the product substitution and developed a fuzzy single-period inventory model for two-item with one-way substitution policy. The above models did not consider the benefits of the cooperation between the retailer and manufacturer.

Recently, the concept of the supply chain management for the fuzzy newsboy problem has triggered considerable attention. Ji and Shao [11] formulated a bilevel programming model for the newsboy problem with fuzzy demands and quantity discounts including multiple retailers. The wholesale prices of newspaper and the ordering quantities of newspaper are decided by maximizing their own expected profits. Xu and Zhai [23] developed a model for fuzzy newsboy problem in the supply chain environment with one manufacturer and one retailer. Xu and Zhai [22] extended the Xu and Zhai's [23] model by developing the decision models to determine the optimal profit for both retailer and manufacturer by coordination.

In real-life problems, sometimes the linguistic information about the demand often varies randomly from expert to expert. In this case, the values of the random variable are fuzzy numbers and it is in accord with the definition of fuzzy random variables presented by Kwakernaak [13]. For example, we consult many experts randomly in order to evaluate the demand. The estimates from different experts may be "high", "low", "about D", "between D_1 and D_2". Dutta et al. [6] investigated a single-period inventory problem with discrete fuzzy random demand involving imprecise probabilities since the probability of a fuzzy event is a fuzzy number. The optimal order quantity maximizing the fuzzy random profit is achieved by using a graded mean integration representation.

The underlying assumption in the aforementioned models is that 100% of items in an ordered lot are perfect. However, the lot sizes produced or ordered may contain some defective products due to imperfect production process, natural disasters, damage in transit and so on. The imperfect quality has influence on lot sizing policy. Several researchers have constructed models for the single-period problem with defective items. For example, Jamal et al. [10] investigated two single-stage production models in which rework is done under two different operational policies to minimize the total cost. Bacel et al. [1] considered the classical newsboy problem with defective products and multi-cycle EOQ model.

From literature survey, there are few literatures dealing with the continuous fuzzy random newsboy problem with imperfect quality in the supply chain environment. In this paper, we attempt to develop the model for newsboy problem with defective products and fuzzy random demand in the supply chain with one manufacturer and one retailer.

The remainder of this paper is organized as follows. Section 2 presents a brief introduction to the preliminary knowledge about fuzzy theory and fuzzy random theory. Section 3 is for assumptions and notations. Section 4 presents two fuzzy random models for the newsboy problem with imperfect quality in the decentralized and centralized systems. Numerical examples and discussion of results are provided in Section 5. Section 6 summarizes and concludes the paper.

2. Preliminaries

In order to consider the fuzzy randomness of an inventory problem, we need the following definitions and property relative to this study.

Definition 1 (*Dubois and Prade [5]*). A fuzzy number $\widetilde{A}$ is a fuzzy set of the real line R whose membership function $\mu_{\widetilde{A}}(y)$ has the following characteristics with $-\infty \leqslant \underline{a} - \Delta_1 \leqslant \underline{a} \leqslant \bar{a} \leqslant \bar{a} + \Delta_2 \leqslant \infty$:

$$\mu_{\widetilde{A}}(y) = \begin{cases} \mu_L(y), & \underline{a} - \Delta_1 \leqslant y \leqslant \underline{a},\ \Delta_1 \geqslant 0 \\ 1, & \underline{a} \leqslant y \leqslant \bar{a} \\ \mu_R(y), & \bar{a} \leqslant y \leqslant \bar{a} + \Delta_2,\ \Delta_2 \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\mu_L:[\underline{a} - \Delta_1, \underline{a}] \to [0,1]$ is continuous and strictly increasing; $\mu_R : [\bar{a}, \bar{a} + \Delta_2] \to [0,1]$ is continuous and strictly decreasing.

If the fuzzy number $\widetilde{A}$ is a triangular fuzzy number, whose membership is parameterized by a triple $(a - \Delta_1, a, a + \Delta_2)$ as the following:

$$\mu_{\widetilde{A}}(y) = \begin{cases} \frac{y-a+\Delta_1}{\Delta_1}, & a - \Delta_1 \leqslant y \leqslant a \\ \frac{a+\Delta_2-y}{\Delta_2}, & a \leqslant y \leqslant a + \Delta_2 \\ 0, & \text{otherwise} \end{cases}$$

Definition 2 (*Kwakernaak [13]*). Let $(R, \mathfrak{A}, P)$ be the probability space, and X be a random variable on $(R, \mathfrak{A}, P)$ with a probability density function $f(x)$. Fuzzy random variable $\widetilde{X}$ is a mapping from R to a family of fuzzy numbers, i.e., $\widetilde{X} : x \in R \to \widetilde{X}(x) \in F$, where F denotes fuzzy set. Intuitively speaking, fuzzy random variables are the random variables that are valued as fuzzy numbers.

For given $\alpha \in (0,1]$, suppose that the α-cut $\widetilde{X}(x)_\alpha$ of fuzzy number $\widetilde{X}(x)$ is $\widetilde{X}(x)_\alpha = \left[\widetilde{X}(x)^-_\alpha, \widetilde{X}(x)^+_\alpha\right]$; Let $\widetilde{X}^-_\alpha, \widetilde{X}^+_\alpha$ denote the left endpoint and the right endpoint of the α-cut $\widetilde{X}(x)_\alpha$ of $\widetilde{X}(x)$, where $\widetilde{X}^-_\alpha, \widetilde{X}^+_\alpha$ are real-valued random variables, i.e.,

$\widetilde{X}_\alpha^- : x \in R \to \widetilde{X}(x)_\alpha^- \in R, \widetilde{X}_\alpha^+ : x \in R \to \widetilde{X}(x)_\alpha^+ \in R$; Note $\widetilde{X}_\alpha = \left[\widetilde{X}_\alpha^-, \widetilde{X}_\alpha^+\right]$, then $\widetilde{X}_\alpha = \left[\widetilde{X}_\alpha^-, \widetilde{X}_\alpha^+\right]$ is the random interval on probability space $(R, \mathfrak{A}, P)$ for given $\alpha \in (0, 1]$.

Definition 3 (*Kwakernaak [13]*). The fuzzy expectation of the fuzzy random variable $\widetilde{X}$ is defined as

$$E(\widetilde{X}) = \bigcup_{\alpha \in [0,1]} \alpha \left[\int_R \widetilde{X}_\alpha^- dP, \ \int_R \widetilde{X}_\alpha^+ dP\right] \tag{1}$$

Referring to Definition 3, the expectation $E(\widetilde{X})$ of the fuzzy random variable $\widetilde{X}$ is a fuzzy number. Note its α-cut as $[E(\widetilde{X})]_\alpha = \left[[E(\widetilde{X})]_\alpha^-, [E(\widetilde{X})]_\alpha^+\right]$, then the fuzzy expectation of the fuzzy random variable $\widetilde{X}$ is

$$E(\widetilde{X}) = \bigcup_{\alpha \in [0,1]} \alpha [E(\widetilde{X})]_\alpha = \bigcup_{\alpha \in [0,1]} \alpha \left[[E(\widetilde{X})]_\alpha^-, \ [E(\widetilde{X})]_\alpha^+\right] \tag{2}$$

Due to $\widetilde{X}_\alpha^-, \widetilde{X}_\alpha^+$ are real-valued random variables, their respective expectation are $E\left[\widetilde{X}_\alpha^-\right] = \int_R \widetilde{X}_\alpha^- dP$ and $E\left[\widetilde{X}_\alpha^+\right] = \int_R \widetilde{X}_\alpha^+ dP$. Then the Eq. (1) can also be expressed as

$$E(\widetilde{X}) = \bigcup_{\alpha \in [0,1]} \alpha \left[E[\widetilde{X}_\alpha^-], \ E[\widetilde{X}_\alpha^+]\right] \tag{3}$$

Based on Eqs. (2) and (3), we have the following inference.

Inference 1. $[E(\widetilde{X})]_\alpha^- = E[\widetilde{X}_\alpha^-]; [E(\widetilde{X})]_\alpha^+ = E[\widetilde{X}_\alpha^+]$.

Inference 1 shows that the α-cut $[E(\widetilde{X})]_\alpha = [[E(\widetilde{X})]_\alpha^-, \ [E(\widetilde{X})]_\alpha^+]$ of fuzzy expectation $E(\widetilde{X})$ can be obtained by calculating the expectation of random endpoints $\widetilde{X}_\alpha^-, \widetilde{X}_\alpha^+$ of random interval $\left[\widetilde{X}_\alpha^-, \widetilde{X}_\alpha^+\right]$.

When the probability density function of random variable X is $f(x)$, Eq. (1) can be expressed as

$$E(\widetilde{X}) = \bigcup_{\alpha \in [0,1]} \alpha \left[\int_R \widetilde{X}_\alpha^-(x) f(x) dx, \ \int_R \widetilde{X}_\alpha^+(x) f(x) dx\right] \tag{4}$$

There exist various defuzzification methods to obtain the estimation of the fuzzy expectation of the fuzzy random variable, such as centroid method, signed distance [24], graded mean integration representation [4]. In this paper, we adopt the widely used signed distance defuzzification method. The same method has been utilized in many researches (e.g. [2,3,16]).

Definition 4 (*Yao and Wu [24]*). Let $\widetilde{A}$ be a fuzzy number with α-cut $\widetilde{A}_\alpha = \left[\widetilde{A}_\alpha^-, \widetilde{A}_\alpha^+\right]$, therefore, the signed distance of fuzzy number $\widetilde{A}$ is $d(\widetilde{A}, 0) = \frac{1}{2}\int_0^1 \left(\widetilde{A}_\alpha^- + \widetilde{A}_\alpha^-\right) d\alpha$.

According to Definition 4, the signed distance of fuzzy expectation $E(\widetilde{X})$ in Eq. (1) is

$$d(E(\widetilde{X}), 0) = \frac{1}{2}\int_0^1 \left[E\left[\widetilde{X}_\alpha^-\right] + E\left[\widetilde{X}_\alpha^+\right]\right] d\alpha, \tag{5}$$

where $E\left[\widetilde{X}_\alpha^-\right] = \int_R \widetilde{X}_\alpha^-(x) f(x) dx, E\left[\widetilde{X}_\alpha^+\right] = \int_R \widetilde{X}_\alpha^+(x) f(x) dx$.

3. Notation and assumptions

The following notation is used:

- w manufacturer's wholesale price per unit;
- c manufacturer's production cost per unit;
- h retailer's holding cost per unit;
- s retailer's shortage cost per unit;
- p retail price per unit;
- b defective rate;
- P_e manufacturer's repurchasing price of defective products;
- f retailer's inspecting cost per unit;
- e the residual value of defective products per unit, where $p > w > c > P_e > e$;
- Q retailer's order quantity;
- Q^* retailer's optimal order quantity in the decentralized system;
- Q^{**} retailer's optimal order quantity in the centralized system;
- X random external demand with probability density function $f(x)$;
- $\widetilde{X}$ fuzzy random external demand corresponding to X and it is valued as the triangular fuzzy number $\widetilde{X}(x) = (x - \Delta_1, x, x + \Delta_2)$, where Δ_1 and Δ_2 are determined by managers depending on their experiences and reflect a kind of fuzzy apperception from their intrinsic understanding;
- x^+ maximum value of x and 0, i.e., $x^+ = \max(x, 0)$.

The mathematical models presented in this study have the following assumptions:

1. A single-period supply chain problem with one manufacturer and one retailer is considered, in which the manufacturer produces and sells goods to the retailer and the retailer faces the external fuzzy random demand from consumers.
2. Only one item is considered.
3. In the decentralized system, at the very beginning of the decision horizon, the manufacturer determines a wholesale price for his items and the retailer makes order policy to maximize its own profit according to the estimate of the external demand and the manufacturer's wholesale price. Once the retailer's order policy is made, the manufacturer must supply the quantity ordered by the retailer. Moreover, there are a proportion of defective items among the items ordered. The retailer has to inspect all the items ordered which incurs substantial cost and the defective items have the residual value.
4. In the centralized system, the manufacturer and the retailer are willing to make a cooperative policy to maximize their joint profit. The defective items inspected are repurchased by the manufacturer at a discounted price and have the same residual value for the manufacturer as that for the retailer in the decentralized system.

4. Model formulation

4.1. Supply chain inventory model in the decentralized system

In the decentralized decision-making situation, the retailer determines order quantity to maximize his profit and the manufacturer makes production plan according to the retailer's order quantity. There is no cooperation between the manufacturer and the retailer. First, the retailer's decision model is formulated. When the retailer is to order Q units, there will be $Q(1-b)$ units of perfect items and the fuzzy random sales volume, fuzzy random holding quantity and fuzzy random shortage quantity for the retailer will be $\min\{\widetilde{X}, Q(1-b)\}$, $(Q(1-b)-\widetilde{X})^+$, and $(\widetilde{X}-Q(1-b))^+$ respectively. Consequently, the retailer's fuzzy random profit consists of fuzzy random sales revenue, fuzzy random holding cost, fuzzy random shortage cost, purchase cost, inspecting cost and the residual value of defective products, which can be expressed as

$$\widetilde{P}_1(Q) = p\min\{\widetilde{X}, Q(1-b)\} - h(Q(1-b)-\widetilde{X})^+ - s(\widetilde{X}-Q(1-b))^+ - wQ + bQe - fQ \tag{6}$$

Furthermore, considering $\min\{\widetilde{X}, Q(1-b)\}$ and $(Q(1-b)-\widetilde{X})^+$ can be expressed as follows:

$$\min\{\widetilde{X}, Q(1-b)\} = \widetilde{X} - (\widetilde{X}-Q(1-b))^+ \tag{7}$$

$$(Q(1-b)-\widetilde{X})^+ = Q(1-b) - \widetilde{X} + (\widetilde{X}-Q(1-b))^+ \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (6), the fuzzy random profit $\widetilde{P}_1(Q)$ is reduced to

$$\widetilde{P}_1(Q) = (p+h)\widetilde{X} - (p+h+s)(\widetilde{X}-Q(1-b))^+ - hQ(1-b) - wQ + bQe - fQ \tag{9}$$

The fuzzy expectation corresponding to fuzzy random profit $\widetilde{P}_1(Q)$ is

$$E(\widetilde{P}_1(Q)) = (p+h)E(\widetilde{X}) - (p+h+s)E((\widetilde{X}-Q(1-b))^+) - hQ(1-b) - wQ + bQe - fQ, \tag{10}$$

where $E(\widetilde{X})$ and $E((\widetilde{X}-Q(1-b))^+)$ are fuzzy expectations of fuzzy random demand and fuzzy random shortage quantity respectively.

Now, from Definition 3 and Eq. (5), we employ the method of signed distance to defuzzify $E(\widetilde{C}(t))$. We get the estimate of the profit in the fuzzy sense as follows:

$$d(E(\widetilde{P}_1(Q)),0) = (p+h)d(E(\widetilde{X}),0) - (p+h+s)d(E((\widetilde{X}-Q(1-b))^+),0) - hQ(1-b) - wQ + bQe - fQ, \tag{11}$$

where $d(E(\widetilde{X}),0)$ and $d(E((\widetilde{X}-Q(1-b))^+),0)$ are the estimate of fuzzy expectations $E(\widetilde{X})$ and $E((\widetilde{X}-Q(1-b))^+)$ respectively.

In order to obtain the specific expression of $d(E(\widetilde{P}_1(Q)),0)$, we must get the function form of $d(E(\widetilde{X}),0)$ and $d(E((\widetilde{X}-Q(1-b))^+),0)$. Referring to Eq. (5), we get

$$d(E(\widetilde{X}),0) = \frac{1}{2}\int_0^1 \left[E\left[(\widetilde{X})^-_\alpha\right] + E\left[(\widetilde{X})^+_\alpha\right]\right]d\alpha = E(X) + \frac{\Delta_2-\Delta_1}{4} \tag{12}$$

$$d(E((\widetilde{X}-Q(1-b))^+),0) = \frac{1}{2}\int_0^1 \left[E\left[((\widetilde{X}-Q(1-b))^+)^-_\alpha\right] + E\left[((\widetilde{X}-Q(1-b))^+)^+_\alpha\right]\right]d\alpha \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (11), we have

$$\begin{aligned} d(E(\widetilde{P}_1(Q)),0) = {} & (p+h)\left(E(X) + \frac{\Delta_2-\Delta_1}{4}\right) - \frac{p+h+s}{2}\int_0^1 \left[E\left[((\widetilde{X}-Q(1-b))^+)^-_\alpha\right] + E\left[((\widetilde{X}-Q(1-b))^+)^+_\alpha\right]\right]d\alpha \\ & - hQ(1-b) - wQ + bQe - fQ \end{aligned} \tag{14}$$

For fixed order quantity Q, the specific function forms of the expectations $E\left[((\widetilde{X}-Q(1-b))^+)^-_\alpha\right]$ and $E[((\widetilde{X}-Q(1-b))^+)^+_\alpha$, which correspond to random variables $((\widetilde{X}-Q(1-b))^+)^-_\alpha$ and $((\widetilde{X}-Q(1-b))^+)^+_\alpha$ respectively, are confirmed by the relationship between the value x of fuzzy random $\widetilde{X}(x)$ and the number of perfect items $Q(1-b)$.

Thus, for fixed order quantity Q, according to the following four situations that the number of perfect items $Q(1-b)$ fells into intervals $x\in(-\infty,Q(1-b)-\Delta_2]$, $[Q(1-b)-\Delta_2\,,Q(1-b)]$, $[Q(1-b),Q(1-b)+\Delta_1]$ and $[Q(1-b)+\Delta_1,+\infty)$, we give the expressions of $E\left[((\widetilde{X}-Q(1-b))^+)^-_\alpha\right]$ and $E[((\widetilde{X}-Q(1-b))^+)^+_\alpha$.

For fuzzy value $\widetilde{X}(x)$ of fuzzy random demand $\widetilde{X}$, the fuzzy shortage quantity is $(\widetilde{X}(x)-Q(1-b))^+$. Denote $\widetilde{Y}(x)=(\widetilde{X}(x)-Q(1-b))^+$.

Case 1. $x\in(-\infty,Q(1-b)-\Delta_2]$. The fuzzy shortage quantity $\widetilde{Y}(x)$ is shown in Fig. 1. For given $\alpha\in(0,1]$, obviously, the interval of random shortage quantity is

$$\left[((\widetilde{X}(x)-Q(1-b))^+)^-_\alpha,\ ((\widetilde{X}(x)-Q(1-b))^+)^+_\alpha\right]=[0,0]\quad x\in(-\infty,Q(1-b)-\Delta_2]$$

The expectation of the random interval above is

$$\left[E\left[((\widetilde{X}(x)-Q(1-b))^+)^-_\alpha\right],\ E\left[((\widetilde{X}(x)-Q(1-b))^+)^+_\alpha\right]\right]=[0,0]$$

Case 2. $x\in[Q(1-b)-\Delta_2,Q(1-b)]$. The fuzzy shortage quantity $\widetilde{Y}(x)$ is shown in Fig. 2.

The membership function of fuzzy shortage quantity $\widetilde{Y}(x)$ is $\mu_{\widetilde{Y}(x)}(t)=\begin{cases}(x-Q(1-b)+\Delta_2-t)/\Delta_2 & 0\leqslant t\leqslant x-Q(1-b)+\Delta_2\\ 0 & \text{other}\end{cases}$, with α-cut is $\left[\widetilde{Y}(x)^-_\alpha,\widetilde{Y}(x)^+_\alpha\right]=\begin{cases}[0,x-Q(1-b)+\Delta_2-\alpha\Delta_2] & 0\leqslant\alpha\leqslant(x-Q(1-b)+\Delta_2)/\Delta_2\\ [0,0] & (x-Q(1-b)+\Delta_2)/\Delta_2\leqslant\alpha\leqslant 1\end{cases}$.

Thus, for given $\alpha\in(0,1]$, the interval of random shortage quantity is

$$\left[((\widetilde{X}(x)-Q(1-b))^+)^-_\alpha,((\widetilde{X}(x)-Q(1-b))^+)^+_\alpha\right]=\begin{cases}[0,0] & x\in[Q(1-b)-\Delta_2,Q(1-b)-\Delta_2+\alpha\Delta_2]\\ [0,x-Q(1-b)+\Delta_2-\alpha\Delta_2] & x\in[Q(1-b)-\Delta_2+\alpha\Delta_2,Q(1-b)]\end{cases}$$

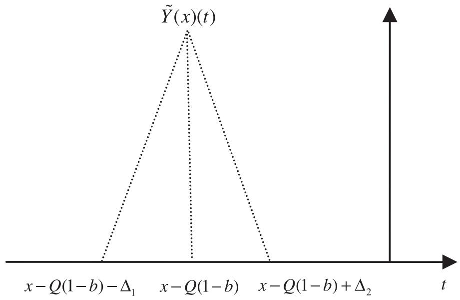

Fig. 1. Fuzzy shortage quantity $\widetilde{Y}(x)$ when $x\in(-\infty\,,Q(1-b)-\Delta_2]$.

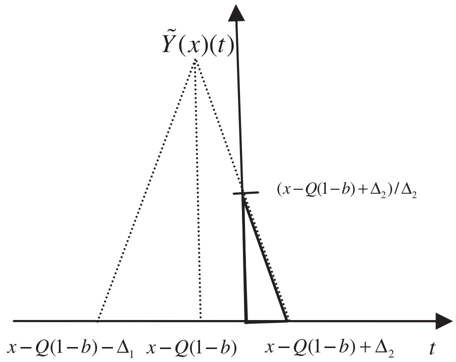

Fig. 2. Fuzzy shortage quantity $\widetilde{Y}(x)$ when $x\in[Q(1-b)-\Delta_2,Q(1-b)]$.

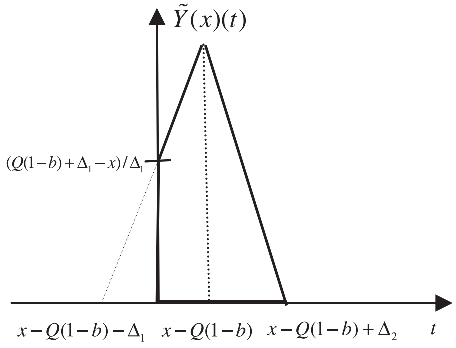

Fig. 3. Fuzzy shortage quantity $\tilde{Y}(x)$ when $x \in [Q(1-b), Q(1-b)+\Delta_1]$.

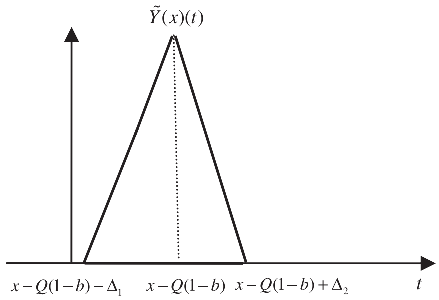

Fig. 4. Fuzzy shortage quantity $\tilde{Y}(x)$ when $x \in [Q(1-b)+\Delta_1, +\infty)$.

The expectation of the random interval above is

$$\left[E\left[((\tilde{X}(x)-Q(1-b))^+)^-_\alpha\right],\ E\left[((\tilde{X}(x)-Q(1-b))^+)^+_\alpha\right]\right] = \begin{cases} [0,0] \\ \left[0, \int_{Q(1-b)-\Delta_2+\alpha\Delta_2}^{Q(1-b)} (x-Q(1-b)+\Delta_2-\alpha\Delta_2) f(x)dx\right] \end{cases}$$

Case 3. $x \in [Q(1-b), Q(1-b)+\Delta_1]$. The fuzzy shortage quantity $\tilde{Y}(x)$ is shown in Fig. 3.

The membership function of fuzzy shortage quantity $\tilde{Y}(x)$ is $\mu_{\tilde{Y}(x)}(t) = \begin{cases} (x-Q(1-b)+\Delta_2-t)/\Delta_2 & x-Q(1-b) \leqslant t \leqslant x-Q(1-b)+\Delta_2 \\ (t-x+Q(1-b)+\Delta_1)/\Delta_1 & 0 \leqslant t \leqslant x-Q(1-b) \end{cases}$, with α-cut is $\left[\tilde{Y}(x)^-_\alpha, \tilde{Y}(x)^+_\alpha\right] = \begin{cases} [0, x-Q(1-b)+\Delta_2-\alpha\Delta_2] & 0 \leqslant \alpha \leqslant (Q(1-b)+\Delta_1-x)/\Delta_1 \\ [x-Q(1-b)-\Delta_1+\alpha\Delta_1, x-Q(1-b)+\Delta_2-\alpha\Delta_2] & (Q(1-b)+\Delta_1-x)/\Delta_1 \leqslant \alpha \leqslant 1 \end{cases}$.

Thus, for given $\alpha \in (0,1]$, the interval of random shortage quantity is

$$\left[((\tilde{X}-Q(1-b))^+)^-_\alpha, ((\tilde{X}-Q(1-b))^+)^+_\alpha\right] = \begin{cases} [0, x-Q(1-b)+\Delta_2-\alpha\Delta_2] & x \in [Q(1-b), Q(1-b)+\Delta_1-\alpha\Delta_1] \\ [x-Q(1-b)-\Delta_1+\alpha\Delta_1, x-Q(1-b)+\Delta_2-\alpha\Delta_2] & x \in [Q(1-b)+\Delta_1-\alpha\Delta_1, Q(1-b)+\Delta_1] \end{cases}$$

The expectation of the random interval above is

$$\left[E\left[((\tilde{X}(x)-Q(1-b))^+)^-_\alpha\right], E\left[((\tilde{X}(x)-Q(1-b))^+)^+_\alpha\right]\right]$$
$$= \begin{cases} \left[0, \int_{Q(1-b)}^{Q(1-b)+\Delta_1-\alpha\Delta_1} (x-Q(1-b)+\Delta_2-\alpha\Delta_2) f(x)dx\right] \\ \left[\int_{Q(1-b)+\Delta_1-\alpha\Delta_1}^{Q(1-b)+\Delta_1} (x-Q(1-b)-\Delta_1+\alpha\Delta_1) f(x)dx, \int_{Q(1-b)+\Delta_1-\alpha\Delta_1}^{Q(1-b)+\Delta_1} (x-Q(1-b)+\Delta_2-\alpha\Delta_2) f(x)dx\right] \end{cases}$$

Case 4. $x \in [Q(1-b)+\Delta_1, +\infty)$. The fuzzy shortage quantity $\widetilde{Y}(x)$ is shown in Fig. 4.

The membership function of fuzzy shortage quantity $\widetilde{Y}(x)$ is $\mu_{\widetilde{Y}(x)}(t) = \begin{cases} (t-x+Q(1-b)+\Delta_1)/\Delta_1 & x-Q(1-b)-\Delta_1 \leqslant t \leqslant x-Q(1-b) \\ (x-Q(1-b)+\Delta_2-t)/\Delta_2 & x-Q(1-b) \leqslant t \leqslant x-Q(1-b)+\Delta_2 \end{cases}$, with α-cut is $\left[\widetilde{Y}(x)^-_\alpha, \widetilde{Y}(x)^+_\alpha\right] = [x-Q(1-b)-\Delta_1+\alpha\Delta_1, x-Q(1-b)+\Delta_2-\alpha\Delta_2]$, $0 \leqslant \alpha \leqslant 1$.

Thus, for given $\alpha \in (0,1]$, the interval of random shortage quantity is

$$\left[((\widetilde{X}-Q(1-b))^+)^-_\alpha, ((\widetilde{X}-Q(1-b))^+)^+_\alpha\right] = [x-Q(1-b)-\Delta_1+\alpha\Delta_1, x-Q(1-b)+\Delta_2-\alpha\Delta_2] \quad x \in [Q(1-b)+\Delta_1, +\infty)$$

The expectation of the random interval above is

$$\begin{aligned} &\left[E\left[((\widetilde{X}(x)-Q(1-b))^+)^-_\alpha\right],\ E\left[((\widetilde{X}(x)-Q(1-b))^+)^+_\alpha\right]\right] \\ &= \left[\int_{Q(1-b)+\Delta_1}^{+\infty} (x-Q(1-b)-\Delta_1+\alpha\Delta_1)f(x)dx, \int_{Q(1-b)+\Delta_1}^{+\infty} (x-Q(1-b)+\Delta_2-\alpha\Delta_2)f(x)dx\right] \end{aligned}$$

Substituting the specific expressions of $\left[E\left[((\widetilde{X}-Q(1-b))^+)^-_\alpha\right]\right.$ and $\left.E\left[((\widetilde{X}-Q(1-b))^+)^+_\alpha\right]\right]$ on $(-\infty, Q(1-b)-\Delta_2]$, $[Q(1-b)-\Delta_2, Q(1-b)]$, $[Q(1-b), Q(1-b)+\Delta_1]$ and $[Q(1-b)+\Delta_1, +\infty)$ into Eq. (14), we have

$$\begin{aligned} d(E(\widetilde{P}_1(Q)),0) &= (p+h)\left(E(X)+\frac{\Delta_2-\Delta_1}{4}\right) - hQ(1-b) - wQ + bQe - fQ \\ &\quad - \frac{p+h+s}{2}\int_0^1 \left[\int_{Q(1-b)-\Delta_2+\alpha\Delta_2}^{Q(1-b)} (x+\Delta_2-\alpha\Delta_2-Q(1-b))f(x)dx \right. \\ &\quad + \int_{Q(1-b)}^{Q(1-b)+\Delta_1-\alpha\Delta_1} (x+\Delta_2-\alpha\Delta_2-Q(1-b))f(x)dx \\ &\quad + \int_{Q(1-b)+\Delta_1-\alpha\Delta_1}^{Q(1-b)+\Delta_1} (x-\Delta_1+\alpha\Delta_1-Q(1-b)+x+\Delta_2-\alpha\Delta_2-Q(1-b))f(x)dx \\ &\quad \left. + \int_{Q(1-b)+\Delta_1}^{+\infty} (x-\Delta_1+\alpha\Delta_1-Q(1-b)+x+\Delta_2-\alpha\Delta_2-Q(1-b))f(x)dx \right] d\alpha \end{aligned}$$

After simplification, the profit for the retailer can be described as below

$$\begin{aligned} d(E(\widetilde{P}_1(Q)),0) &= (p+h)\left(E(X)+\frac{\Delta_2-\Delta_1}{4}\right) - hQ(1-b) - wQ + bQe - fQ \\ &\quad - \frac{(p+h+s)}{2}\left[\int_0^1 \int_{Q(1-b)+\Delta_1-\alpha\Delta_1}^{+\infty} (x-\Delta_1+\alpha\Delta_1-Q(1-b))f(x)dx\,d\alpha \right. \\ &\quad \left. + \int_0^1 \int_{Q(1-b)-\Delta_2+\alpha\Delta_2}^{+\infty} (x+\Delta_2-\alpha\Delta_2-Q(1-b))f(x)dx\,d\alpha \right] \end{aligned} \tag{15}$$

Proposition 1

(1) *$d(E(\widetilde{P}_1(Q)),0)$ is strictly concave with respect to Q.*
(2) *The optimal order quantity Q^* satisfies*

$$\frac{1}{2}\int_0^1 F(Q^*(1-b)-\Delta_2+\alpha\Delta_2) + F(Q^*(1-b)+\Delta_1-\alpha\Delta_1)d\alpha = \frac{(p+s)(1-b)-w+be-f}{(p+h+s)(1-b)}. \tag{16}$$

Proof

(1) We take the first and the second derivatives of Eq. (15) with respect to Q and obtain

$$\frac{d[d(E(\widetilde{P}_1(Q)),0)]}{dQ} = -(p+h+s)\left\{\frac{1-b}{2}\int_0^1 [F(Q(1-b)-\Delta_2+\alpha\Delta_2)+F(Q(1-b)+\Delta_1-\alpha\Delta_1)]d\alpha - (1-b)\right\} - w + be - f - h(1-b)$$

$$\frac{d^2[d(E(\widetilde{P}_1(Q)),0)]}{dQ^2} = -\frac{(p+h+s)(1-b)^2}{2}\int_0^1 [f(Q(1-b)-\Delta_2+\alpha\Delta_2)+f(Q(1-b)+\Delta_1-\alpha\Delta_1)]d\alpha$$

For given $\alpha \in [0,1]$, we have $\frac{d^2[d(E(\widetilde{P}(Q)),0)]}{dQ^2} < 0$ since $f(Q(1-b)-\Delta_2+\alpha\Delta_2) \geqslant 0$ and $f(Q(1-b)+\Delta_1-\alpha\Delta_1) > 0$. That is, the $d(E(\widetilde{P}_1(Q)),0)$ is strictly concave with respect to Q.

Here comes the result.

(2) We set $\frac{d[d(E(\widetilde{P}_1(Q)),0)]}{dQ} = 0$ and obtain the condition which the optimal order quantity Q^* satisfies as follows:

$$\frac{1}{2}\int_0^1 F(Q^*(1-b) - \Delta_2 + \alpha\Delta_2) + F(Q^*(1-b) + \Delta_1 - \alpha\Delta_1)d\alpha = \frac{(p+s)(1-b) - w + be - f}{(p+h+s)(1-b)}$$

The proof is completed. □

Then, the manufacturer's profit model is formulated. If the retailer's optimal order quantity is Q^*, the expression of the corresponding profit for manufacturer is as below:

$$M_1(Q^*) = (w-c)Q^* \tag{17}$$

Therefore, in the decentralized system, the whole supply chain profit is

$$T_1(Q^*) = d(E(\widetilde{P}_1(Q^*)),0) + M_1(Q^*) \tag{18}$$

4.2. Supply chain inventory model in centralized system

In this section, we studied the centralized decision-making model in which the manufacturer and the retailer are willing to cooperate to get their optimal joint profit in order to improve the performance of the whole supply chain. In this paper, the manufacturer repurchases the defective products at a discounted price. The retailer's order quantity and the scope of the manufacturer's repurchasing price are derived by maximizing the total profit for the whole supply chain.

The retailer's profit can be expressed as

$$\widetilde{P}_2(Q) = (p+h)\widetilde{X} - (p+h+s)(\widetilde{X} - Q(1-b))^+ - hQ(1-b) - wQ + bQP_e - fQ \tag{19}$$

The manufacturer's profit can be written as

$$M_2(Q) = (w-c)Q - bQP_e + bQe \tag{20}$$

Thus, since the Eq. (19) and Eq. (20), the fuzzy random whole supply chain profit is expressed as follows

$$\widetilde{T}(Q) = \widetilde{P}_2(Q) + M_2(Q) = (p+h)\widetilde{X} - (p+h+s)(\widetilde{X} - Q(1-b))^+ - hQ(1-b) - cQ + bQe - fQ \tag{21}$$

The fuzzy expectation corresponding to fuzzy random whole supply chain profit $\widetilde{T}(Q)$ is

$$E(\widetilde{T}(Q)) = E(\widetilde{P}_2(Q)) + M_2(Q) = (p+h)E(\widetilde{X}) - (p+h+s)E((\widetilde{X} - Q(1-b))^+) - hQ(1-b) - cQ + bQe - fQ, \tag{22}$$

where $E(\widetilde{X})$ and $E((\widetilde{X} - Q(1-b))^+)$ are fuzzy expectations of fuzzy random demand and fuzzy random shortage quantity respectively.

Now, from Definition 3 and Eq. (5), we employ the method of signed distance to defuzzify $E(\widetilde{C}(t))$. We get the estimate of the profit in the fuzzy sense as follows:

$$d(E(\widetilde{T}(Q)),0) = (p+h)d(E(\widetilde{X}),0) - (p+h+s)d(E((\widetilde{X} - Q(1-b))^+),0) - hQ(1-b) - cQ + bQe - fQ, \tag{23}$$

where $d(E(\widetilde{X}),0)$ and $d(E((\widetilde{X} - Q(1-b))^+),0)$ are the estimate of fuzzy expectations $E(\widetilde{X})$ and $E((\widetilde{X} - Q(1-b))^+)$ respectively.

Denote $T_2(Q) = d(E(\widetilde{T}(Q)),0)$. Then, the problem can be reduced to the following optimization problem $\max_{Q\geqslant 0} T_2\,(Q)$.

Proposition 2

(1) *$T_2(Q)$ is strictly concave with respect to Q.*
(2) *The optimal order quantity Q^{**} satisfies*

$$\frac{1}{2}\int_0^1 F(Q^{**}(1-b) - \Delta_2 + \alpha\Delta_2) + F(Q^{**}(1-b) + \Delta_1 - \alpha\Delta_1)d\alpha = \frac{(p+s)(1-b) - c + be - f}{(p+h+s)(1-b)} \tag{24}$$

Proof

(1) Taking the first and the second derivatives of Eq. (15) with respect to Q, we have

$$\frac{d[T_2(Q)]}{dQ} = -(p+h+s)\left\{\frac{1-b}{2}\int_0^1 [F(Q(1-b) - \Delta_2 + \alpha\Delta_2) + F(Q(1-b) + \Delta_1 - \alpha\Delta_1)]d\alpha - (1-b)\right\} - c + be - f - h(1-b)$$

$$\frac{d^2[T_2(Q)]}{dQ^2} = -\frac{(p+h+s)(1-b)^2}{2}\int_0^1 [f(Q(1-b) - \Delta_2 + \alpha\Delta_2) + f(Q(1-b) + \Delta_1 - \alpha\Delta_1)]d\alpha$$

For given $\alpha \in [0,1]$, we have $\frac{d^2[T_2(Q)]}{dQ^2} < 0$ since $f(Q(1-b) - \Delta_2 + \alpha\Delta_2) \geqslant 0$ and $f(Q(1-b) + \Delta_1 - \alpha\Delta_1) > 0$. Hence, $T_2(Q)$ is strictly concave with respect to Q.

(2) Setting $\frac{d[T_2(Q)]}{dQ} = 0$, we obtain the condition which the optimal order quantity Q^{**} satisfies as follows

$$\frac{1}{2}\int_0^1 F(Q^{**}(1-b) - \Delta_2 + \alpha\Delta_2) + F(Q^{**}(1-b) + \Delta_1 - \alpha\Delta_1)d\alpha = \frac{(p+s)(1-b) - c + be - f}{(p+h+s)(1-b)}$$

It is proved completely. □

Proposition 3. *The optimal order quantity in centralized system is larger than that in decentralized system, that is $Q^* < Q^{**}$.*

Proof. Based on the Propositions 1 and 2, we have

$$\frac{1}{2}\int_0^1 F(Q^{*}(1-b) - \Delta_2 + \alpha\Delta_2) + F(Q^{*}(1-b) + \Delta_1 - \alpha\Delta_1)d\alpha = \frac{(p+s)(1-b) - w + be - f}{(p+h+s)(1-b)}$$
$$\frac{1}{2}\int_0^1 F(Q^{**}(1-b) - \Delta_2 + \alpha\Delta_2) + F(Q^{**}(1-b) + \Delta_1 - \alpha\Delta_1)d\alpha = \frac{(p+s)(1-b) - c + be - f}{(p+h+s)(1-b)}$$

Due to $w > c$, we have $\frac{(p+s)(1-b)-w+be-f}{(p+h+s)(1-b)} < \frac{(p+s)(1-b)-c+be-f}{(p+h+s)(1-b)}$. Besides, the distribution function $F(x) = \int_{-\infty}^{x} f(t)dt$ is a monotonic increasing function of x. Hence, $Q^* < Q^{**}$ is derived. The proof is completed. □

Proposition 4. *The whole supply chain profit in centralized system is larger than that in decentralized system, that is $T_2(Q^{**}) > T_1(Q^*)$.*

Proof. Comparing Eq. (11) with Eq. (23), the whole supply chain profit in centralized system can be expressed as

$$\begin{aligned}
T_2(Q) &= d(E(\widetilde{T}(Q)),0) = d(E(\widetilde{P}_2(Q)),0) + M_2(Q) \\
&= (p+h)\left(E(X) + \frac{\Delta_2 - \Delta_1}{4}\right) - hQ(1-b) - cQ + bQe - fQ \\
&\quad - \frac{(p+h+s)}{2}\left[\int_0^1 \int_{Q(1-b)+\Delta_1-\alpha\Delta_1}^{+\infty} (x - \Delta_1 + \alpha\Delta_1 - Q(1-b))f(x)dx\,d\alpha \right. \\
&\quad \left. + \int_0^1 \int_{Q(1-b)-\Delta_2+\alpha\Delta_2}^{+\infty} (x + \Delta_2 - \alpha\Delta_2 - Q(1-b))f(x)dx\,d\alpha\right]
\end{aligned}$$

Referring to Eqs. (15) and (17), the whole supply chain profit in decentralized system is

$$\begin{aligned}
T_1(Q) &= d(E(\widetilde{P}_1(Q)),0) + M_1(Q) \\
&= (p+h)\left(E(X) + \frac{\Delta_2 - \Delta_1}{4}\right) - hQ(1-b) - cQ + bQe - fQ \\
&\quad - \frac{(p+h+s)}{2}\left[\int_0^1 \int_{Q(1-b)+\Delta_1-\alpha\Delta_1}^{+\infty} (x - \Delta_1 + \alpha\Delta_1 - Q(1-b))f(x)dx\,d\alpha \right. \\
&\quad \left. + \int_0^1 \int_{Q(1-b)-\Delta_2+\alpha\Delta_2}^{+\infty} (x + \Delta_2 - \alpha\Delta_2 - Q(1-b))f(x)dx\,d\alpha\right]
\end{aligned}$$

We note that $T_1(Q)$ and $T_2(Q)$ have the same mathematical expression. Denote $TT(Q) = T_1(Q) = T_2(Q)$. Although the mathematical expression of $T_1(Q)$ is the same as $T_2(Q)$ in the two decision systems, the optimal order quantities determined by different decision-making mechanism are different. Moreover, according to Proposition 2, the optimal order quantity maximizing the $TT(Q)$ is Q^{**} and $TT(Q)$ is concave with respect to Q. $TT(Q^*) < TT(Q^{**})$ can be obtained because of $Q^* < Q^{**}$ which results from Proposition 3. □

Proposition 3 shows that centralized decision-making mechanism can stimulate the purchasing. Proposition 4 suggests that the whole supply chain profit in centralized system is larger than that in decentralized system. It reveals that a cooperative policy between the retailer and the manufacturer is optimal for achieving the increase in the whole supply chain profit. This paper incorporates the manufacturer's repurchasing policy for the defective products in order to make increase in the retailer and the manufacturer's respective profit and promote the cooperation. The scope of the repurchasing price is determined as well.

Proposition 6. *The manufacturer's repurchasing price satisfies*

$$P_e \geqslant \left\{ -\frac{(p+h+s)}{2}\left[\int_0^1 \int_{Q^*(1-b)+\Delta_1-\alpha\Delta_1}^{+\infty} (x-\Delta_1+\alpha\Delta_1-Q^*(1-b))f(x)dx\,d\alpha \right.\right.$$
$$- \int_0^1 \int_{Q^{**}(1-b)+\Delta_1-\alpha\Delta_1}^{+\infty} (x-\Delta_1+\alpha\Delta_1-Q^{**}(1-b))f(x)dx\,d\alpha \int_0^1 \int_{Q^*(1-b)-\Delta_2+\alpha\Delta_2}^{+\infty} (x+\Delta_2-\alpha\Delta_2-Q^*(1-b))f(x)dx\,d\alpha$$
$$\left. - \int_0^1 \int_{Q^{**}(1-b)-\Delta_2+\alpha\Delta_2}^{+\infty} (x+\Delta_2-\alpha\Delta_2-Q^{**}(1-b))f(x)dx\,d\alpha \right]$$
$$\left. - w(Q^*-Q^{**}) + beQ^* - f(Q^*-Q^{**}) - h(Q^*-Q^{**})(1-b) \right\}/(bQ^{**}) \quad \text{and} \quad P_e \leqslant [bQ^{**}e-(w-c)(Q^*-Q^{**})]/(bQ^{**}).$$

Proof. In the decentralized system, the manufacturer's profit is $M_1(Q^*) = (w-c)Q^*$ and the retailer's profit is $d(E(\widetilde{P}_1(Q^*)),0)$. While in the centralized system, the manufacturer's profit is $M_2(Q^{**}) = (w-c)Q^{**} - bQ^{**}P_e + bQ^{**}e$ and the retailer's profit is $d(E(\widetilde{P}_2(Q^{**})),0)$.

The manufacturer and the retailer are willing to cooperate only if their respective profit in the centralized system is larger than that in the decentralized system. Hence, in order to ensure the cooperation, the manufacturer's profit and the retailer's profit must satisfy

$$\begin{cases} d(E(\widetilde{P}_1(Q^*)),0) \leqslant d(E(\widetilde{P}_2(Q^{**})),0) \\ M_1(Q^*) \leqslant M_2(Q^{**}) \end{cases}$$

Similar to

$$d(E(\widetilde{P}_1(Q^*)),0) = (p+h)\left(E(X)+\frac{\Delta_2-\Delta_1}{4}\right) - wQ^* + bQ^*e - fQ^* - hQ^*(1-b)$$
$$-\frac{(p+h+s)}{2}\left[\int_0^1 \int_{Q^*(1-b)+\Delta_1-\alpha\Delta_1}^{+\infty} (x-\Delta_1+\alpha\Delta_1-Q^*(1-b))f(x)dx\,d\alpha\right.$$
$$\left. + \int_0^1 \int_{Q^*(1-b)-\Delta_2+\alpha\Delta_2}^{+\infty} (x+\Delta_2-\alpha\Delta_2-Q^*(1-b))f(x)dx\,d\alpha \right],$$

we have

$$d(E(\widetilde{P}_2(Q^{**})),0) = (p+h)\left(E(X)+\frac{\Delta_2-\Delta_1}{4}\right) - wQ^{**} + bQ^{**}P_e - fQ^{**} - hQ^{**}(1-b)$$
$$-\frac{(p+h+s)}{2}\left[\int_0^1 \int_{Q^{**}(1-b)+\Delta_1-\alpha\Delta_1}^{+\infty} (x-\Delta_1+\alpha\Delta_1-Q^{**}(1-b))f(x)dx\,d\alpha\right.$$
$$\left. + \int_0^1 \int_{Q^{**}(1-b)-\Delta_2+\alpha\Delta_2}^{+\infty} (x+\Delta_2-\alpha\Delta_2-Q^{**}(1-b))f(x)dx\,d\alpha \right].$$

Therefore, we obtain

$$d(E(\widetilde{P}_1(Q^*)),0) - d(E(\widetilde{P}_2(Q^{**})),0) = -\frac{(p+h+s)}{2}\left[\int_0^1 \int_{Q^*(1-b)+\Delta_1-\alpha\Delta_1}^{+\infty} (x-\Delta_1+\alpha\Delta_1-Q^*(1-b))f(x)dx\,d\alpha\right.$$
$$\left. - \int_0^1 \int_{Q^{**}(1-b)+\Delta_1-\alpha\Delta_1}^{+\infty} (x-\Delta_1+\alpha\Delta_1-Q^{**}(1-b))f(x)dx\,d\alpha \right]$$
$$-\frac{(p+h+s)}{2}\left[\int_0^1 \int_{Q^*(1-b)-\Delta_2+\alpha\Delta_2}^{+\infty} (x+\Delta_2-\alpha\Delta_2-Q^*(1-b))f(x)dx\,d\alpha\right.$$
$$\left. - \int_0^1 \int_{Q^{**}(1-b)-\Delta_2+\alpha\Delta_2}^{+\infty} (x+\Delta_2-\alpha\Delta_2-Q^{**}(1-b))f(x)dx\,d\alpha \right]$$
$$- w(Q^*-Q^{**}) + beQ^* - bP_eQ^{**} - f(Q^*-Q^{**}) - h(Q^*-Q^{**})(1-b) \leqslant 0$$

Moreover, we have $M_1(Q^*) - M_2(Q^{**}) = (w-c)(Q^*-Q^{**}) + bQ^{**}P_e - bQ^{**}e \leqslant 0$.

After the simplification of the above inequalities, here comes the result. □

Proposition 6 reveals that the lowest value for the retailer's profit in the centralized system is obtained for the lowest value of the parameter P_e and this value coincides with the result obtained in the decentralized system. In the case of the manufacturer's profit in the centralized system, the lowest value is obtained for the highest value of P_e and it's the same

Table 1
Variation of Δ_1, Δ_2 and b effects on the optimal solutions.

Δ_1	Δ_2	b	Q^*	$M_1(Q^*)$	$d(E(\widetilde{P}_1(Q^*)),0)$	$T_1(Q^*)$	Q^{**}	$T_2(Q^{**})$	$T_2(Q^{**})-T_1(Q^*)$	P_e
10	10	0.03	58.89	294.45	385.91	680.36	60.93	685.44	5.08	$7.80 \leqslant P_e \leqslant 10.58$
		0.05	59.99	299.95	366.10	666.05	62.11	671.34	5.29	$6.71 \leqslant P_e \leqslant 8.41$
		0.07	61.12	305.60	345.49	651.09	63.35	656.66	5.57	$6.26 \leqslant P_e \leqslant 7.51$
		0.09	62.31	311.55	324.02	635.57	64.63	641.36	5.79	$6.00 \leqslant P_e \leqslant 6.99$
8	10	0.03	59.52	297.60	403.52	701.12	61.22	705.30	4.18	$7.35 \leqslant P_e \leqslant 9.63$
		0.05	60.66	303.30	383.50	686.80	62.43	691.13	4.33	$6.45 \leqslant P_e \leqslant 7.84$
		0.07	61.84	309.20	362.65	671.85	63.68	676.37	4.52	$6.05 \leqslant P_e \leqslant 7.06$
		0.09	63.07	315.35	340.94	656.29	64.99	660.99	4.70	$5.84 \leqslant P_e \leqslant 6.64$
5	10	0.03	60.65	303.25	428.62	731.87	61.79	734.31	2.44	$6.76 \leqslant P_e \leqslant 8.07$
		0.05	61.86	309.30	408.25	717.55	63.03	720.01	2.46	$6.07 \leqslant P_e \leqslant 6.86$
		0.07	63.11	315.55	387.03	702.58	64.33	705.11	2.53	$5.79 \leqslant P_e \leqslant 6.35$
		0.09	64.41	322.05	364.91	686.96	65.67	689.57	2.61	$5.62 \leqslant P_e \leqslant 6.07$
5	12	0.03	60.68	303.40	423.76	727.16	61.87	729.73	2.57	$6.82 \leqslant P_e \leqslant 8.21$
		0.05	61.88	309.40	403.38	712.78	63.11	715.41	2.63	$6.11 \leqslant P_e \leqslant 6.95$
		0.07	63.14	315.70	382.14	697.84	64.40	700.49	2.65	$5.81 \leqslant P_e \leqslant 6.40$
		0.09	64.44	322.20	360.01	682.21	65.75	684.93	2.72	$5.65 \leqslant P_e \leqslant 6.11$
5	14	0.03	60.70	303.50	418.86	722.36	61.94	725.05	2.69	$6.89 \leqslant P_e \leqslant 8.34$
		0.05	61.91	309.55	398.47	708.02	63.18	710.72	2.70	$6.16 \leqslant P_e \leqslant 7.01$
		0.07	63.16	315.80	377.23	693.03	64.47	695.78	2.75	$5.84 \leqslant P_e \leqslant 6.45$
		0.09	64.46	322.30	355.09	677.39	65.81	680.21	2.82	$5.66 \leqslant P_e \leqslant 6.14$

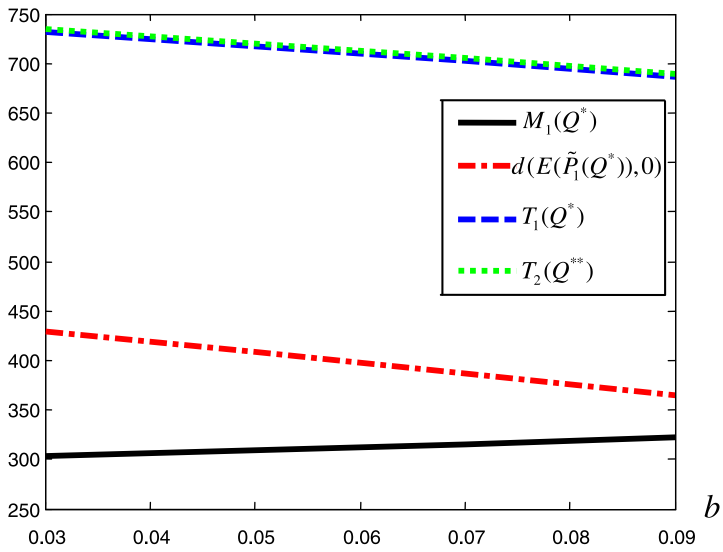

Fig. 5. Variation of defective rate b effects on the profits $d(E(\widetilde{P}_1(Q^*)),0)$, $M_1(Q^*)$, $T_1\ (Q^*)$ and $T_2(Q^{**})$.

result under the decentralized system. Note that the value of the parameter P_e should be generally selected lager than residual value of detective products per unit and less than the manufacturer's production cost per unit. Sometimes, the value of the parameter P_e can be determined larger than the manufacturer's production cost per unit, because it can stimulate the demand of the retailer. In this way, the loss for the repurchasing items may be less than the profit for the additional items.

5. Numerical examples

In order to illustrate the effectiveness of the above solution procedure, let us consider an inventory system with the following data, many of which is used in Xu and Zhai [23]: the retail's sale price per unit $p = 30$, retailer's holding cost per unit $h = 12$, retailer's shortage cost per unit $s = 10$, manufacturer's wholesale price per unit $w = 20$, manufacturer's production cost per unit $c = 15$. Besides, the residual value of detective products per unit $e = 5$, the detective rate $b = 0.05$, the retailer's inspecting cost per unit is $f = 1$, the external demand has a normal probability density function $f(x)$ with finite mean $\mu = 60$ and standard deviation $\sigma = 2$, and it is valued as $\widetilde{X}(x) = (x-5, x, x+10)$.

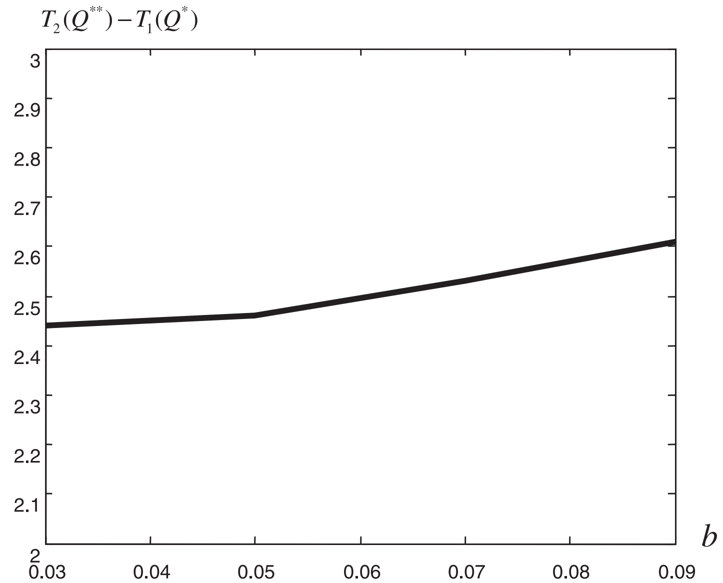

Fig. 6. Variation of defective rate b effects on the difference between $T_1(Q^*)$ and $T_2(Q^{**})$.

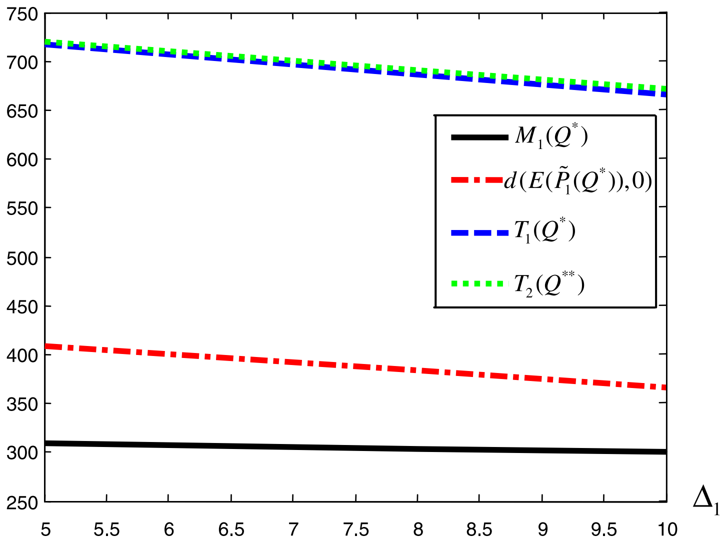

Fig. 7. Variation of Δ_1 effects on the profits $d(E(\widetilde{P}_1(Q^*)),0)$, $M_1(Q^*)$, $T_1(Q^*)$ and $T_2(Q^{**})$.

Situation 1. In the decentralized system, based on Proposition 2, we note that the optimal order quantity can be derived from

$$\frac{1}{2}\int_0^1 F(Q^*(1-b) - \Delta_2 + \alpha\Delta_2) + F(Q^*(1-b) + \Delta_1 - \alpha\Delta_1)d\alpha = \frac{(p+s)(1-b) - w + be - f}{(p+h+s)(1-b)}$$

Unfortunately, the optimal order quantity Q^* cannot be expressed in a closed form. Therefore, using the integral transform method, the above equation can be transformed to

$$\begin{aligned}
&\frac{Q(1-b)-\mu+\Delta_1}{\Delta_1}\Phi\left(\frac{Q(1-b)-\mu+\Delta_1}{\sigma}\right) + \left(\frac{Q(1-b)-\mu}{\Delta_2} - \frac{Q(1-b)-\mu}{\Delta_1}\right)\Phi\left(\frac{Q(1-b)-\mu}{\sigma}\right) \\
&\quad - \frac{Q(1-b)-\mu-\Delta_2}{\Delta_2}\Phi\left(\frac{Q(1-b)-\mu-\Delta_2}{\sigma}\right) + \frac{\sigma}{\Delta_1}\phi\left(\frac{Q(1-b)-\mu-\Delta_1}{\sigma}\right) + \left(\frac{\sigma}{\Delta_2} - \frac{\sigma}{\Delta_1}\right)\phi\left(\frac{Q(1-b)-\mu}{\sigma}\right) \\
&\quad - \frac{\sigma}{\Delta_2}\phi\left(\frac{Q(1-b)-\mu-\Delta_2}{\sigma}\right) = \frac{2[(p+s)(1-b) - w + be - f]}{(p+h+s)(1-b)},
\end{aligned} \tag{25}$$

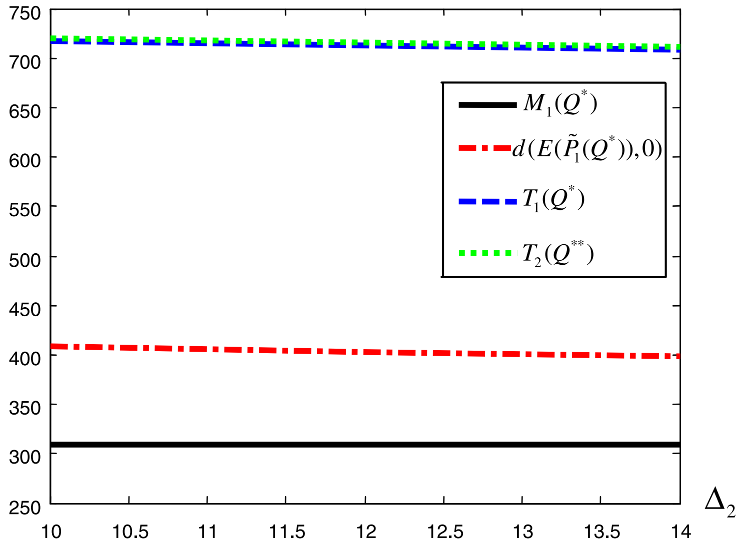

Fig. 8. Variation of Δ_2 effects on the profits $d(E(\widetilde{P}_1(Q^*)), 0)$, $M_1(Q^*)$, $T_1(Q^*)$ and $T_2(Q^{**})$.

where the $\phi(x)$ and $\Phi(x)$ are the probability density function and cumulative distribution function of the standard normal distribution respectively.

Referring to the Eq. (25), in the decentralized system, the retailer's optimal order quantity is $Q^* \approx 62$. Based on Eqs. (15), (17) and (18), the retailer's profit is $d(E(\widetilde{P}_1(Q^*)), 0) \approx 408$, the manufacturer's profit is $M_1(Q^*) \approx 309$ and the whole supply chain profit is $T_1(Q^*) \approx 717$.

Situation 2. In the centralized system, similar to the discussion in situation 1, the retailer's optimal order quantity is $Q^{**} \approx 64$. The whole supply chain profit is $T_2(Q^{**}) \approx 720 > T_1(Q^*)$. By Proposition 6, the scope of the manufacturer's repurchasing price is $6.07 \leqslant P_e \leqslant 6.86$.

This paper analyzes the effect of defective rate and the fuzzy randomness of demand on the optimal order quantity in the centralized and decentralized systems, the whole supply chain profit in the centralized and decentralized systems, the repurchasing price in the centralized system and the difference between the whole supply chain profit in the centralized and decentralized systems, which is shown in Table 1.

Table 1 reveals that: (1) As the defective rate increases, there is increase in the optimal order quantity as well as the manufacturer's profit but there is decrease in the retailer's profit as well as the whole supply chain profit in the decentralized system. In the centralized system, the increase in the defective rate causes larger optimal order quantity but the lower the whole supply chain profit and the lower the bounds of the repurchasing price. Variation of the defective rate effects on the profits is also depicted in Fig. 5. (2) A lower value of Δ_1 or a higher value of Δ_2, namely, a hike in demand, causes larger optimal quantity in both the decentralized system and the centralized system, which implies the demand can stimulate the purchase. (3) A higher value of Δ_1 or Δ_2, namely, the increase in the uncertainty of demand, causes lower retailer's profit, and lower the whole supply chain profit in the decentralized system. In this case, lower the whole supply chain profit and higher the bounds of the repurchasing price occurred in the centralized system. The more uncertainty of demand, the more holding cost and shortage cost the retailer will bear, and the less profit the retailer will obtain in the decentralized system. As the value of Δ_1 increases or the value of Δ_2 decreases, namely, the demand decreases, the retailer's optimal order quantity decreases and the manufacturer's profit drops slightly in the decentralized system. Fig. 7 exhibits the effects of varying Δ_1 on the profits. Variation of Δ_2 effects on the profits is shown in Fig. 8. (4) When the defective rate increases the difference between $T_2(Q^{**})$ and $T_1(Q^*)$ increases. Furthermore, variation of the defective rate effects on the difference between $T_2(Q^{**})$ and $T_1(Q^*)$ is presented in Fig. 6. It reveals that the cooperative policy in the centralized system was productive.

6. Conclusions

This paper dealt with the newsboy problem with imperfect quality in the supply chain environment. Two models with external fuzzy random demand in the decentralized system and centralized system are developed. The optimal solutions for the two models are determined and analyzed contrastively. A cooperative policy is applied to facilitate the optimization of the whole supply chain profit. Future research can be done for considering multi-product newsboy problem with imperfect quality in the supply chain environment.

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